

Weighing the universe with accelerators and detectors

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Abstract

Suppose the lightest superpartner (LSP) is observed at colliders, and WIMPs are detected in explicit experiments. We point out that one cannot immediately conclude that cold dark matter (CDM) of the universe has been observed, and we determine what measurements are necessary before such a conclusion is meaningful. We discuss the analogous situation for neutrinos and axions; in the axion case we have not found a way to conclude axions are the CDM even if axions are detected.

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I. INTRODUCTION

Despite the many successes of the standard cosmology, we still do not know the composition and the amount of energy density in our universe (for a review, see for example [1]). The existence of matter which does not emit much light is certain from the fact that stars and other luminous matter contribute only a tiny fraction (about 0.004) [2] of the critical energy density (required for a spatially flat universe) while the orbits of stars around galaxies (see e.g. [3]) indicate that the gravitating energy density is about ten times larger.¹ From a cosmological point of view, a variety of observations, such as the peculiar motions of galaxies as well as the masses of clusters of galaxies, corroborate the existence of dark energy and indicate that Ω_M , the ratio of the average matter-energy density in our Hubble volume to the critical energy density, is about 1/3 if the Hubble expansion rate is about 70 km/s/Mpc as the current observations indicate (see e.g. [4]). Although without other constraints this dark energy density could be in the form of ordinary baryonic matter (objects made of neutrons and protons), the standard model of big bang nucleosynthesis (BBN) calculations and the measurement of primeval abundance of deuterium gives strong evidence that the baryon density is $\Omega_B h^2 = (0.02 \pm 0.002)$ which implies Ω_B of about 0.04. Hence, most of the matter energy density in the universe seems to be in the form of nonbaryonic dark matter (NBDM).

Measurements of cosmological parameters will determine Ω_{NBDM} accurately, but are unlikely to help us know the actual character of the NBDM, since they mainly depend on its gravitational interactions. Some information on NBDM collisional energy loss and clustering will also come from astrophysics information. For perhaps the most likely forms of NBDM we can hope to observe the actual particles in laboratory experiments, and calculate to a few percent accuracy the actual contribution to Ω [5].

Remarkably, the more likely extensions to the standard model of particle physics provide candidates for the nonbaryonic dark matter, candidates which existed even before the need for NBDM was established. Indeed, given that these extensions to the standard model are theoretically compelling from particle physics considerations, without cosmological considerations, their provision of dark matter energy of the right order of magnitude abundance provides an independent hint at the existence of physics beyond the standard model. Among the various candidates for this nonbaryonic dark matter (see e.g. [1]), a typical unified supersymmetric theory with conserved R-parity will have a stable lightest supersymmetric particle (LSP) that will probably constitute most of the cold dark matter (CDM). In addition, the non-baryonic dark energy would most likely consist of neutrino hot dark matter (ν HDM), axionic CDM (ACDM), and the cosmological constant (Λ). Given a Lagrangian for the unified theory (UT) and thermal equilibrium initial conditions for the fluid determining the

¹In this paper, we will use the standard notation of ratio of energy density in X to the critical energy density as Ω_X . Since the critical energy density is determined by the Hubble expansion rate which is uncertain, we will use the usual parameterization $H = h100$ km/s/Mpc and often write the energy density as $\Omega_X h^2$ value instead.

cosmological evolution,² one can calculate the dark matter content of our universe by solving the Boltzmann equations.

Many other NBDM candidates have been proposed [6]. In this paper we will only analyze the situations for neutrinos, axions, and LSPs in some detail. Similar conclusions hold for the rest. At appropriate places in the paper we will consider non-thermal evolution.

The neutrino situation is simpler. Once the cosmology is known and the neutrino masses are determined, one can compute the relic number of neutrinos, multiply by the masses, and obtain Ω_ν . If neutrinos have anomalous interactions they have to be included.

The axion case is difficult. Assume axions are observed in terrestrial detectors, and perhaps even information about them comes from astrophysical data. Then the thermal density $\Omega_{a(t)}$ can be computed precisely enough [7]. However, coherent oscillations of axion zero modes give non-thermal contributions $\Omega_{a(nt)}$ that depend strongly on a “misalignment” factor, related to the value of the axion field at the confinement transition. We are aware of no way to determine $\Omega_{a(nt)}$, which can be the dominant contribution, so we conclude that it may not be possible to even know how much of the CDM is $\Omega_{axion} = \Omega_{a(t)} + \Omega_{a(nt)}$.

The LSP case turns out to be difficult, but solvable. We first demonstrate that knowing the LSP mass, and even knowing its cross section on a nucleon, do not allow one to know its contribution to Ω_{LSP} to better accuracy than an order of magnitude or so. Part of this uncertainty arises from a current lack of knowledge of the phases of the soft supersymmetry breaking Lagrangian (\mathcal{L}_{soft}), but even if one arbitrarily took all phases to be zero or π at least a factor of six uncertainty in Ω_{LSP} remains. And for an important question such as the composition of the CDM, one would not want to make unwarranted assumptions about the phases or other relevant parameters. We then show that knowing the relevant parameters of \mathcal{L}_{soft} , and $\tan\beta$, to about 5% allows a determination of Ω_{LSP} to similar accuracy. Such measurements will be possible by combining data from hadron colliders, low energy (*e.g.* electric dipole moments) experiments, B-factories, and a lepton collider with a polarized beam and sufficient energy to produce several superpartners (and appropriate luminosity).

If WIMPs are observed in explicit (*i.e.*, non-collider) detection experiments (direct underground WIMP scattering on nuclei, or space-based, or indirectly via ν interactions from WIMP annihilation in the sun or earth) there are additional large uncertainties which we do not consider in this paper. These include going from WIMP-nucleus to WIMP-quark cross sections, local density and velocity distributions of WIMPs, how long antiprotons or positrons can persist in the galaxy, and so on. All of our results hold even if these other factors could be controlled.

In the next two sections we discuss the LSP case in detail, first analytically and then numerically. Then we turn to some cosmological considerations, and examine the axion case. In all cases we assume that h^2 will be known accurately before it is needed to achieve a particle physics knowledge of Ω_{NBDM} , so we do not include errors in h^2 . We also assume

²The cosmological initial conditions can be determined by an inflationary model, which in principle can also be determined by the UT if the initial conditions for inflation are set by an even more fundamental principle or UT offers a unique inflationary history. We assume that the underlying cosmology is known when the relic abundances are finally calculated.

that the usual Boltzmann treatment is sufficient [8] to estimate the possible uncertainties and that refinements of the calculational procedure necessary for more accurate calculation can be accomplished when appropriate. For examples of such refinements, see Ref. [9] and references therein.

II. ANALYTIC ESTIMATES

In this section, we give an analytic estimate for the uncertainties that can be expected in the calculation of LSP CDM, given that some of the parameters in the MSSM has been measured to a certain accuracy. The most common situation for most parts of the MSSM parameter space is when no other particle mass is within $m_\chi/20$ of the LSP mass m_χ [10,11]. In that case, self-annihilation determines the relic abundance. Assuming as usual that the annihilation products are in chemical thermal equilibrium with the massless degrees of freedom in a radiation dominated flat FRW universe, the simplified Boltzmann equation governing the relic abundance can be written as

$$\frac{df}{dx} = m_\chi \sqrt{\frac{45M_{pl}^2}{4\pi^3 g_*}} \langle \sigma v \rangle (f^2 - f_0^2) \quad (1)$$

where $f = n_\chi/T^3$ is the LSP volume density scaled by the cube of the temperature which sets the scale for the volume density of the photons, $\langle \sigma v \rangle$ is the thermal averaged self annihilation cross section, $x = T/m_\chi$ is the scaled temperature, g_* counts the degrees of freedom contributing to the entropy, and $f_0 = \frac{x^{-3/2}}{\sqrt{2\pi^2}} e^{-1/x}$ is the nonrelativistic approximation of the thermal equilibrium volume density of LSPs. Starting from a thermal equilibrium initial conditions, f tracks f_0 until the freeze-out temperature x_F and then f decouples from f_0 . Approximating this decoupling to occur sharply at a particular temperature x_F , one finds an expression for the relic density as

$$\Omega = T_X^3 \sqrt{\frac{4\pi^3 g_*}{45M_{pl}^2}} \left(\int_{x_0}^{x_F} \langle \sigma v \rangle dx \right)^{-1} / \rho_c \quad (2)$$

where ρ_c is the critical energy density that only depends on the cosmology. The approximate expression for the freezeout temperature can be written as

$$x_F \approx \frac{1}{\ln[m_\chi \xi \langle \sigma v \rangle] + \frac{1}{2} \ln x_F}$$

$$\xi \equiv \frac{1}{(2\pi)^3} \sqrt{\frac{45M_{pl}^2}{2g_*}}$$

Propagating the error in quadratures, the fractional error is characterized by

$$\left\langle \left(\frac{\Delta \Omega}{\Omega} \right)^2 \right\rangle = \sum_i (\Delta_i (\delta P_i))^2$$

where δP_i denotes the uncertainty in parameter P_i . Neglecting the T and g_* uncertainties, we can write

$$\Delta_i = \frac{[x_F^2 \frac{\partial m_\chi}{\partial P_i} \frac{\sqrt{g_*}}{m_\chi} \langle \sigma v \rangle + x_F^2 \sqrt{g_*} \frac{\partial \langle \sigma v \rangle}{\partial P_i} - \int \frac{\partial \langle \sigma v \rangle}{\partial P_i} \sqrt{g_*} dx]}{\int_{x_0}^{x_F} \sqrt{g_*} \langle \sigma v \rangle dx}$$

where we have used the fact that $x_F \ll 1$. We can approximate $\frac{\partial \langle \sigma v \rangle}{\partial P_i} \approx \langle \sigma v \rangle \frac{r_i}{P_i}$ where $r_i \sim O(1)$ (which is a good approximation in the region of the parameter space where the P_i dependence is analytic) and conclude

$$\Delta_i \sim \frac{r_i}{P_i}.$$

Hence, we conclude that

$$\langle \left(\frac{\Delta \Omega}{\Omega} \right)^2 \rangle = \sum_i r_i^2 \left(\frac{\delta P_i}{P_i} \right)^2$$

where the strength of the error contribution is determined by $r_i = \frac{\partial \ln \langle \sigma v \rangle}{\partial \ln P_i}$ which is what we expect.

For example, consider a typical nonresonant self-annihilation cross section of an LSP going to two fermions through a sfermion exchange. We have

$$\langle \sigma v \rangle \approx \frac{g_{f\tilde{f}\chi}^2}{64\pi} \sqrt{1 - \frac{m_f^2}{m_\chi^2}} \left[\frac{c_1 m_f^2 + c_2 x m_\chi^2}{(m_{\tilde{f}}^2 + m_\chi^2 - m_f^2)^2} \right]$$

where c_1 and c_2 are $O(1)$ dimensionless constants and $g_{f\tilde{f}\chi}^2$ is the fermion-sfermion-neutralino coupling. Then considering the contribution of m_χ to the fractional error, we find to leading order in $\frac{m_f}{m_\chi} \sim \frac{m_f}{m_{\tilde{f}}}$,

$$r_{m_\chi} = \frac{2 \left(1 - \left(\frac{m_{\tilde{f}}}{m_\chi} \right)^2 \right)}{1 + \left(\frac{m_{\tilde{f}}}{m_\chi} \right)^2}$$

which is about $\lesssim 1$. Considering the contribution of $m_{\tilde{f}}$ to the uncertainty, we find similarly

$$r_{m_{\tilde{f}}} = \frac{-4}{1 + \left(\frac{m_\chi}{m_{\tilde{f}}} \right)^2}$$

which has a magnitude of about $\gtrsim 2$. Finally we see that the coupling will also contribute

$$r_{g_{f\tilde{f}\chi}} = 2,$$

showing that the abundance is most sensitive to the sfermion mass and the LSP-sfermion-fermion coupling. In addition we see that relative uncertainties in parameters are likely to lead to even larger relative uncertainties in Ω_{LSP} .

The standard scalar neutralino cross section on protons [12] is expressed as

$$\sigma_p^{scalar} = \frac{4m_r^2}{\pi} f_p^2 \quad (3)$$

where the effective scalar interaction coupling f_p is usually dominated by the CP-even Higgs parton level exchange between quarks and neutralinos [13]

$$f_p^{scalar} \simeq f_p^{(H)} = m_p \left[\sum_{q=u,d,s} f_{T_q}^p \frac{f_q^H}{m_q} + \frac{2}{27} f_{TG}^p \sum_{q=c,b,t} \frac{f_q^H}{m_q} \right]. \quad (4)$$

The matrix element coefficients $f_{T_q}^p$ and $f_{TG}^p = 1 - \sum_q f_{T_q}^p$ can be extracted from pion-nucleon scattering using chiral perturbation theory and are subject to large uncertainties which are reflected in the neutralino-proton scattering cross section [14].

The parton level couplings, on the other hand, depend entirely on the SUSY Lagrangian parameters and Higgs masses

$$f_q^H = m_q \sum_{i=1,2} \frac{c_\chi^{(i)} c_q^{(i)}}{m_{H_i}^2}$$

with the (in general complex) neutralino couplings to the light CP-even Higgs

$$c_\chi^{(1)} = \frac{1}{2}(gN_{12}^* - g'N_{11}^*)(N_{13}^* \sin \alpha + N_{14}^* \cos \alpha)$$

and to the heavy Higgs

$$c_\chi^{(2)} = \frac{1}{2}(gN_{12}^* - g'N_{11}^*)(N_{14}^* \sin \alpha - N_{13}^* \cos \alpha),$$

where α is the Higgs mixing angle. The quark-Higgs couplings depend on weak isospin quantum number and we have for the up type quarks

$$c_u^{(1)} = -\frac{g}{2m_W} \frac{\cos \alpha}{\sin \beta}, \quad c_u^{(2)} = -\frac{g}{2m_W} \frac{\sin \alpha}{\sin \beta} \quad (5)$$

and for the down type quarks

$$c_d^{(1)} = \frac{g}{2m_W} \frac{\sin \alpha}{\cos \beta}, \quad c_d^{(2)} = -\frac{g}{2m_W} \frac{\cos \alpha}{\cos \beta}. \quad (6)$$

In the large m_A limit $\alpha \simeq \beta - \pi/2$ and the $c_d^{(2)}$ coupling is enhanced by a factor of $\tan \beta$. As a result, for small values of $\tan \beta$ (less than 4) the cross section is dominated by the light Higgs exchange while for large values of $\tan \beta$ the heavy Higgs exchange contribution prevails.

In analogy to the analysis of the relic density we can determine accuracy of the cross section calculation as a function of the variation in individual parameters

$$\left\langle \left(\frac{\Delta \sigma_p}{\sigma_p} \right)^2 \right\rangle = \sum_i s_i^2 \left(\frac{\Delta P_i}{P_i} \right)^2$$

where s_i is the variation coefficient corresponding to the uncertainty δP_i in parameter P_i . Assuming that there is indeed a dominant contribution from one of the Higgs bosons, we find that the Higgs and quark coupling both contribute to the uncertainty in a similar way

$$s_{c_q} \simeq s_{c_\chi} \simeq 2 \quad (7)$$

and the Higgs mass contribution to the uncertainty is

$$s_{m_H} \simeq -4. \quad (8)$$

while the cross section is largely insensitive to variations of the neutralino mass since the reduced mass of the system is very close to the mass of the proton.

It is important to realize that the neutralino-Higgs couplings depend crucially on the neutralino mixing matrix. Both gaugino and Higgsino components are required to participate if the couplings are not to vanish. The neutralino mass matrix depends on complex quantities M_1 , M_2 and μ with potentially large phases [15] which can significantly modify the lightest neutralino composition and subsequently its couplings to the Higgs bosons. It is important to include these complex phases in the general analysis of the relic density and the neutralino proton cross section in order to be able to estimate the overall uncertainty in these quantities which can be achieved once the SUSY Lagrangian parameters including the phases are measured.

NUMERICAL RESULTS

We illustrate the general behavior of the neutralino relic density and proton elastic cross section on a characteristic set of the MSSM parameters which can provide neutralino relic abundance in the cosmologically relevant region and a cross section small enough to be allowed by direct detection experiments. Since the neutralino scattering cross section grows with $\tan\beta$ models with low and moderate values of $\tan\beta$ can easily satisfy direct detection constraints and still provide a significant neutralino abundance. In all of our numerical calculations we use the following reference set of parameters – $M_1 = 80$ GeV, $m_A = 250$ GeV, $\tan\beta = 3$, $\varphi_1 = \varphi_\mu = 0$, $m_{\tilde{\nu}} = 110$ GeV, $m_{\tilde{\ell}_L} = 125$ GeV, $m_{\tilde{\ell}_R} = 110$ GeV, $m_{\tilde{q}_L} = 420$ GeV, $m_{\tilde{u}_R} = 400$ GeV, $m_{\tilde{d}_R} = 380$ GeV.

This set has been chosen so that the resulting neutralino is predominantly a bino and the effects of the light Higgs and Z pole neutralino annihilation pole as well as any co-annihilation effects are minimized. Our choice of parameters leads to the values of $\Omega h^2 \simeq 0.148$ and $\sigma_p \simeq 11.4 \times 10^{-9} pb$. Since we are working in a general parametric framework, the soft Higgs masses can be chosen so that electroweak symmetry breaking conditions are satisfied. These numbers are not special, and only illustrate typical results.

First let us turn to the discussion of the neutralino observables sensitivity to the CP-conserving parameters appearing in the supersymmetric Lagrangian. Obviously, the most important ones are M_1 , M_2 , μ and $\tan\beta$ which enter into neutralino mass matrix and determine both the mass of the lightest neutralino m_χ and its Higgs and lepton-slepton couplings. Another significant variation comes from the change in the mass of the CP-odd Higgs boson which influences the neutralino-Higgs couplings and the heavy CP-even Higgs

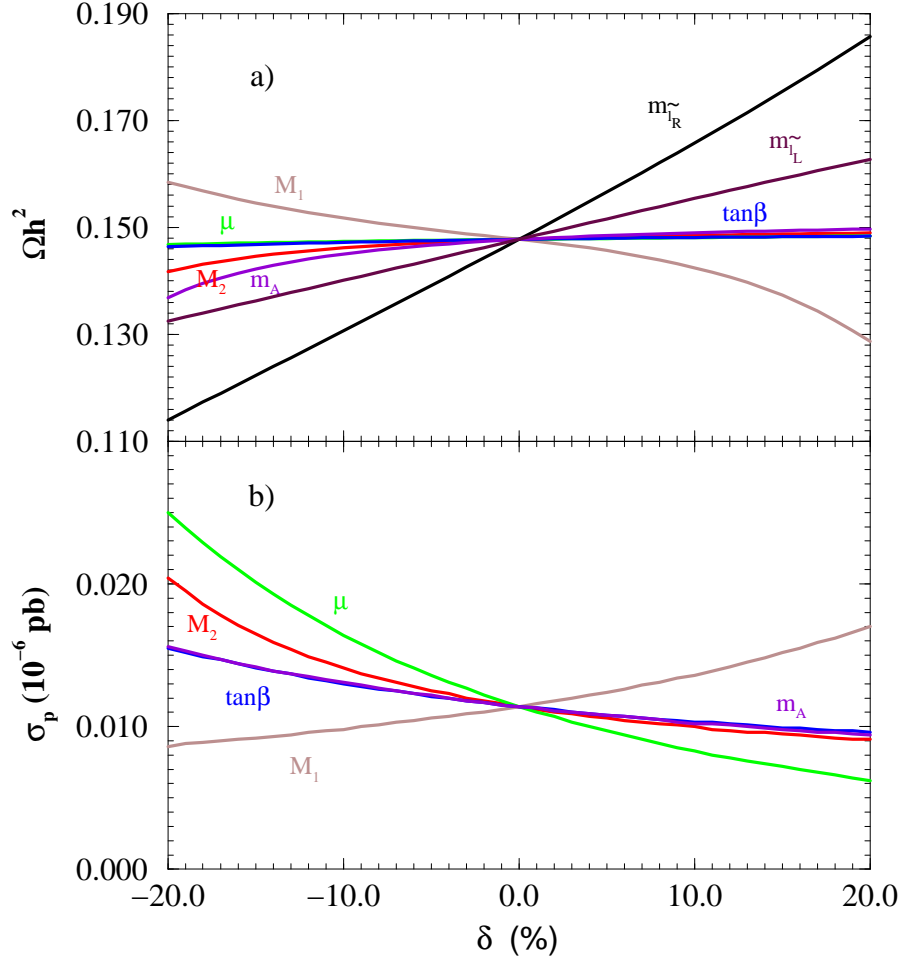


FIG. 1. Variation of the neutralino relic density (a) and of the neutralino-proton elastic scattering cross section (b) with $\delta = \Delta c_i / c_i$ for the most relevant SUSY parameters.

boson mass. The dominant slepton exchange contribution to the neutralino annihilation cross section depends crucially on the slepton mass while the scattering cross section is insensitive to the scalar masses.

Fig. 1 shows the dependence of the neutralino relic density and proton cross section on the relative variation of the important parameters within a $\pm 20\%$ range. As shown in frame a), the relic density is mostly sensitive to variations in the slepton masses appearing in the dominant annihilation diagram – as the slepton mass increases the annihilation cross section is more suppressed and Ωh^2 increases. All the other parameters enter the neutralino sector and their variations are reflected in the variations of the LSP mass and the LSP-slepton-lepton coupling. The most significant is the variation of the density with the bino mass M_1 which determines $m_{\tilde{\chi}}$ and subsequently the dominant s -wave contribution to the neutralino annihilation cross section. It is relatively stable with respect to changes in M_2 , μ and $\tan\beta$ since the annihilation cross section depends weakly on the neutralino composition. In summary, the overall variation of Ωh^2 is at most $\pm 25\%$ for any single parameter in the given range of the input SUSY parameters.

On the other hand, the spin independent part of the neutralino-proton scattering cross section depends crucially on the gaugino-Higgsino composition of the lightest neutralino since both components take part in the neutralino-Higgs interaction. This fact is reflected in frame *b*) which shows a comparable sensitivity to M_1 , M_2 and μ . It is obvious that for the given range of $\delta = \Delta c/c$ the change in σ_p can be as big as a factor of two. Note that in our particular case ($\tan\beta = 3$) the cross section is dominated by the light Higgs boson exchange and consequently the sensitivity to m_A is limited. As $\tan\beta$ increases, the heavy Higgs exchange takes over the cross section and the sensitivity to m_A , which determines the heavy Higgs boson mass, increases correspondingly.

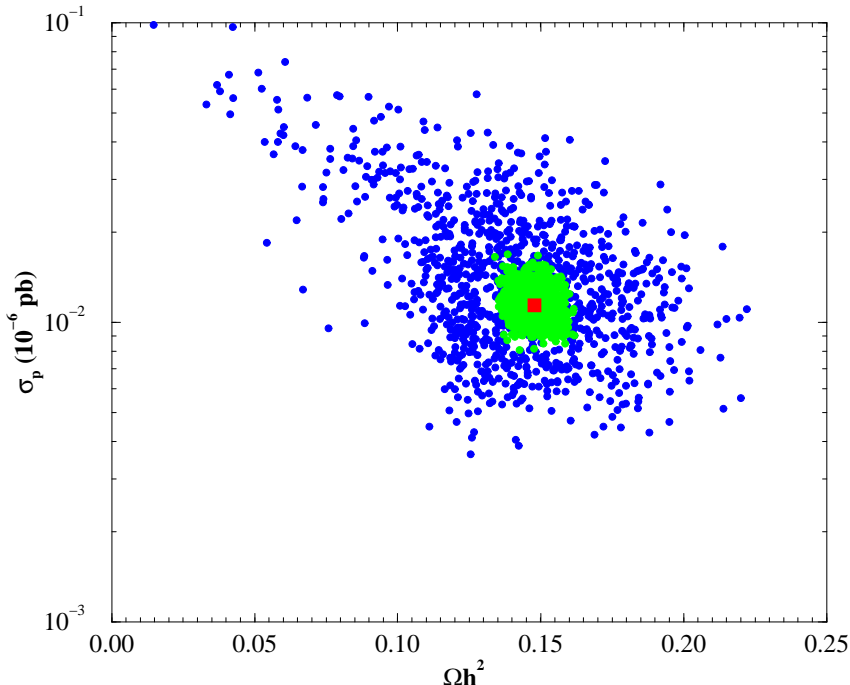


FIG. 2. Plot of regions in the Ωh^2 – σ_p plane resulting from varying all relevant parameters from Fig. 1 around their standard value (square) within 5% (green (grey) region) and 20% (dots). The CP-violating phases are set to zero.

In order to estimate the accuracy with which we can determine the relic density and the proton scattering cross section once we have measured the SUSY parameters at a collider experiment we plot the range of both quantities when the SUSY parameters are all varied within a 20 % range from the central value. Fig. 2 shows the resulting region in the Ωh^2 – σ_p plane. In part of parameter space the variations of Fig. 1 can combine, and lead to much larger ranges of variation in Ωh^2 . In particular, the points in the upper left region with small relic density result from non-linear effects associated with the presence of an s-channel light Higgs exchange annihilation pole, which can drastically increase the annihilation cross section, and parameter points on the tail of the Breit-Wigner resonance have small neutralino relic density. This is a fairly generic situation since the allowed range for neutralino relic

abundance imposes a limit on the neutralino mass. Note that the 5% error region allows a reasonable determination of Ωh^2 . To understand the weak constraints from direct detection experiments, imagine a horizontal line across Fig.2, with a height uncertainty coming from the nuclear physics and astrophysics ambiguities in extracting σ_p , and notice the resulting uncertainties in Ωh^2 .

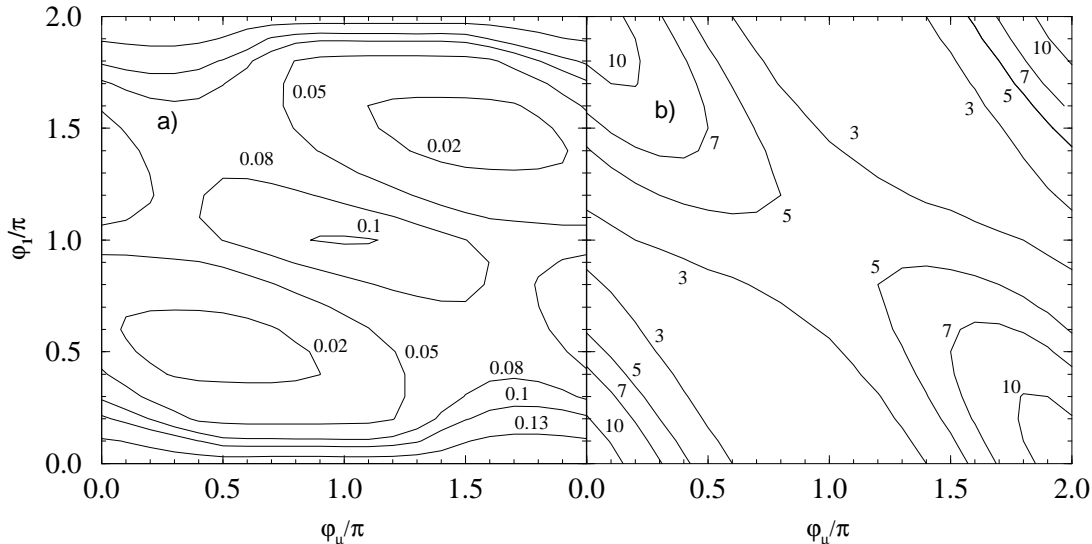


FIG. 3. Variation of the relic abundance Ωh^2 (a) and LSP proton cross section σ_p (b) (in 10^{-6} pb) with CP-violating phases affecting the neutralino mass matrix. All other parameters are set to the standard set values.

Next we examine the effects of soft SUSY phases on these questions. It has been demonstrated recently [15] that all soft SUSY phases can be large. Even if it turned out that some are smaller, *e.g.* from future electric dipole moment experiments, others relevant here may not be. Also, phases have two effects – not only do they directly enter the calculations [16] of Ωh^2 and σ_p , they also make measurement of other parameters such as $\tan \beta$ more difficult, and impossible at LEP or hadron colliders [17].

We work in a phase parametrization consistent with [15] and choose $\varphi_2 = \text{Arg}(M_2) = 0$. In Fig. 3 we show the range of Ωh^2 (frame a)) and σ_p (frame b)) as the two relevant phases entering the neutralino mass matrix φ_1 and φ_μ are varied in their full range while all other parameters are kept constant. It turns out that the relic abundance has two local minima and maxima and the range is bigger than an order of magnitude between the minimum and maximum. The neutralino proton cross section also varies by more than an order of magnitude but it is monotonous in the φ_μ - φ_1 plane. From Fig. 3 it is clear that by neglecting the phases one can be missing a crucial part of the particle physics information needed to determine the LSP abundance.

COSMOLOGICAL UNCERTAINTIES AND AXIONS

Although the focus of our paper is examining the uncertainties in our knowledge of dark matter coming from particle physics, here we will briefly comment on some of the cosmological uncertainties related to the dark matter determination. Specifically, we would like to emphasize a point that can easily be overlooked: only with a firm determination of the cosmological history can the particle physics data weigh the universe. To make this point concrete, let us consider some possible difficulties with the usual approach to the dark matter calculation which we have assumed in our paper. First we will consider the effect of energy density behaving like a cosmological constant today. Then, we will consider the situation when the dominant contribution to zero pressure dark energy is not a thermal relic. In such “nonthermal” cases, particle physics data generically cannot determine the “matter” energy density contribution because “history” or boundary conditions of the particle physics may not be determined by a fundamental principle or dynamics. We will choose axionic dark matter to illustrate this point. Such uncertainties are relevant to all forms of dark matter. There can also be extra entropy production which has to be included.

Let us first consider how the existence of energy density with a negative pressure [18](which we will for simplicity call ρ_Λ) affects the relic LSP density calculations. First, consider the Boltzmann equations. Suppose the spatial curvature is negligible as will be the case after inflation. Then, we can start with the same Boltzmann equation

$$\frac{dn}{dt} + 3Hn = -\langle\sigma v\rangle(n^2 - n_0^2)$$

Since conservation of entropy is still valid, we will have

$$\frac{df}{dt} = -\langle\sigma v\rangle T^3(f^2 - f_0^2)$$

where $f \equiv \frac{n}{T^3}$. In the usual scenario, we assume radiation dominance and use

$$t = \frac{1}{2H}$$

This is still a good approximation because the temperature at which the relic density froze out is $O(\text{GeV}) \gg T_{\text{nucleosynthesis}}$ and we require that ρ_Λ not destroy one of the pillars of standard cosmology and that Ω_Λ be a monotonic function of time. Hence, the rest of the formalism follows as usual for the determination of the freezeout temperature. As far as the evolution of the relic density after freezeout is concerned, there is no change from before again because the nucleosynthesis temperature is much lower than the freezeout temperature.

Of course, if we do not require monotonic variation of ρ_Λ then the above relation between the time and expansion rate may no longer valid and the relic abundance calculation may have to be modified. However, this is unlikely because to preserve the successes of BBN, the models of ρ_Λ then must be fine tuned such that its energy is significant at the time of dark matter freezeout while being negligible at the time of BBN [19]. In the unlikely event that such tuned history is that of our universe, then the dark matter energy density implied by the particle physics measurements may be significantly different from what we have calculated in this paper.

Axion scenarios offer an elegant solution to the strong CP problem. Furthermore, pseudoscalars resembling axions are generically expected [20] from effective theories arising from compactifications of string theories.³ Unlike LSPs, the axions are expected to naturally contribute significantly to the cosmological energy density even when the thermal relic abundance does not contribute significantly. Even with the assumption of inflation and sufficiently low reheating temperature [21] rendering possible axionic topological defect related contributions irrelevant, one must account for the fact that generically there may be coherent oscillations of the axion zero modes (commonly called misalignment contribution [7]) giving a nonthermal contribution to the cosmological energy density with the magnitude

$$\Omega_{a(nt)} h^2 = 0.13 \times 10^{\pm 0.4} \left(\frac{\Lambda_{QCD}}{200 \text{ MeV}} \right)^{-0.7} f(\theta_1) \theta_1^2 \left(\frac{m_a}{10^{-5} \text{ eV}} \right)^{-1.18}$$

where m_a is the axion mass, f is some known monotonically increasing function accounting for anharmonic effects ($f(0) = 1$), and θ_1 is the “amplitude” of the oscillations of the axion field. The value of θ_1 is essentially equal to the value of the axion field at the confinement transition. Because there is no direct way to measure θ_1 , even with a precise determination of m_a , there will be a great uncertainty in the axionic contribution. This is in contrast with the most likely smaller [7] thermal relic contribution of the axion which can be quite precisely determined once the mass of the axion is measured. Indeed, most of the relic axion detection experiments such as electromagnetic spectrum observation of nearby clusters, as well as Sikivie-type resonant microwave cavities immersed in strong magnetic fields, mostly measure the axion mass, and there is no obvious direct handle on the “misalignment” angle θ_1 . Indeed, θ_1 may be most likely determined randomly from the space of all possible values, since above the confinement transition the axions have essentially zero mass. Hence, in such “nonthermal” cosmological boundary condition dependent situations, the determination of the dark matter from particle physics data seems impossible.

CONCLUSION

Understanding the composition of our universe is one of the most fundamental issues we can study. We demonstrate that we can only be fully confident we have learned the answer if we can calculate from first principles and laboratory data for each type of matter X what Ω_X is, and find agreement with cosmological measurement of Ω_{matter} . For neutrinos this can be done if their masses can be measured, or shown to be small compared to $\sim 1 \text{ eV}$. For axions we have emphasized there does not seem to be a way to do this.

For the LSP it is possible to calculate Ω_{LSP} if measurements of soft supersymmetry phases, and $\tan \beta$ are made. Unless such measurements are done, it is simply incorrect to suggest that detection of the LSP at a collider, or in a direct or indirect WIMP experiment, means that the cold dark matter has been observed. Indeed, there is a loose correlation of easier detection with smaller relic density. All measurements, of course, help constrain the

³However, most embeddings of axions in string compactification models run into trouble with scales although there may be some ways to circumvent the problem [20].

situation, and the detection of the LSP will mean that *some* of the CDM has been observed – but perhaps only a fraction.

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